

ECT372	QUANTUM COMPUTING	CATEGORY	L	T	P	CREDIT
		PEC	2	1	0	3

**Preamble:** Quantum computers are not yet built. If such machines become a reality, they will fundamentally change how we perform calculations, and the implications on many applications (including communications and computer security) will be tremendous. This course aims to provide a first introduction to quantum computing with a general understanding of how quantum mechanics can be applied to computational problems. It highlights the paradigm change between conventional computing and quantum computing, and introduce several basic quantum algorithms.

**Prerequisite:** MAT101 Linear Algebra and Calculus

**Course Outcomes:** After the completion of the course the student will be able to

<b>CO 1 K2</b>	Explain the basic constructs in linear algebra needed to build the concepts of quantum computing
<b>CO 2 K2</b>	Relate the postulates of quantum mechanics for computation and illustrate/ demonstrate quantum measurement
<b>CO 3 K3</b>	Identify quantum gates and build quantum circuit model in which most of the quantum algorithms are designed.
<b>CO 4 K4</b>	Analyse and design quantum algorithms and grasp the advantage they offer over classical counterparts.

#### Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO1	3	3										2
CO2	3	3										2
CO3	3	3	3									2
CO4	3	3	3	2								2

#### Assessment Pattern

Bloom's Category		Continuous Assessment Tests		End Semester Examination
		1	2	
Remember	K1	10	10	10
Understand	K2	20	20	20
Apply	K3	20	10	50
Analyse	K4		10	20
Evaluate				
Create				

**Mark distribution**

Total Marks	CIE	ESE	ESE Duration
150	50	100	3 hours

**Continuous Internal Evaluation Pattern:**

Attendance	: 10 marks
Continuous Assessment Test (2 numbers)	: 25 marks
Assignment/Quiz/Course project	: 15 marks

**End Semester Examination Pattern:** There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

**Course Level Assessment Questions****Course Outcome 1 (CO1): Explain the basic constructs in linear algebra needed to build the concepts of quantum computing (K2)**

1. Summarise the basic operators and matrices required for understanding the quantum computing concepts.
2. Find the Eigen values and Eigen vectors of Pauli matrices.
3. Explain spectral decomposition and Spectral theorem. State the spectral theorem for Hermitian operator.
4. Show the matrix representation of the tensor products of the Pauli operators

**Course Outcome 2 (CO2): Relate the postulates of quantum mechanics for computation and illustrate/ demonstrate quantum measurement (K2)**

1. State and explain the postulates of Quantum Mechanics applied to computing.
2. Show the Bloch sphere representation of quantum bits.
3. Find the various states of a given system using state space analysis.
4. Demonstrate the state space representation of composite systems using tensor product.

**Course Outcome 3 (CO3): Identify quantum gates and build quantum circuit model in which most of the quantum algorithms are designed (K3)**

1. Model universal gates using standard quantum gates.
2. Illustrate the implementation of quantum operation using quantum gates.
3. Construct and prove circuit identities.
4. Construct a circuit for implementing controlled U operations
5. Design quantum circuits that implements projective measurement in the computational basis.

**Course Outcome 4 (CO4): Analyse and design quantum algorithms and grasp the advantage they offer over classical counterparts (K4)**

1. Design a circuit that implements Quantum Fourier Transform(QFT) for an n-bit input.
2. Construct the phase estimation algorithm from basic principles and design the circuit for phase estimation using QFT.
3. Interpret phase estimation algorithm for the implementation of order finding and factorisation algorithms.

**SYLLABUS**

**Module 1: Basics of Linear Algebra**

History and Overview of Quantum Computation and Quantum Information, Linear Algebra Basics, Linear Operators and matrices, The Pauli matrices, Inner Products, Eigen values and Eigen vectors, Hermitian operators and Adjoints, Spectral theorem, Tensor Products.

**Module 2: Basics of Quantum Mechanics**

State Space Representation - Bloch Sphere, State Evolution – Unitary transformation, Quantum measurement – Projective measurements, Composite systems - Superposition.

**Module 3: Quantum Gates and Circuits**

Quantum gates – Hadamard gate, NOT gate, controlled-NOT gate, Toffoli gate, Realisation of classical gates with quantum gates – Z Gate, Fredkein Gate, Pauli Matrices – Controlled Swap and Controlled U-operations, Circuit Identities

**Module 4: Quantum Measurement**

Basic principle of quantum measurement - Principle of deferred measurement, Principle of implicit measurement, Gates with projective measurements, Universal quantum gates, Universality of two level unitary gates.

**Module 5: Algorithms**

Quantum Fourier Transform (QFT) – Quantum circuit for QFT, Quantum phase estimation, Modular exponentiation, Order finding and factorisation – Deutsch’s algorithm.

**Text Books**

1. M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, Cambridge, UK, Cambridge University Press, 2010.
2. J. Gruska, Quantum Computing, McGraw Hill, 1999.
3. G. Strang, Linear algebra and its applications (4th Edition), Thomson, 2006.

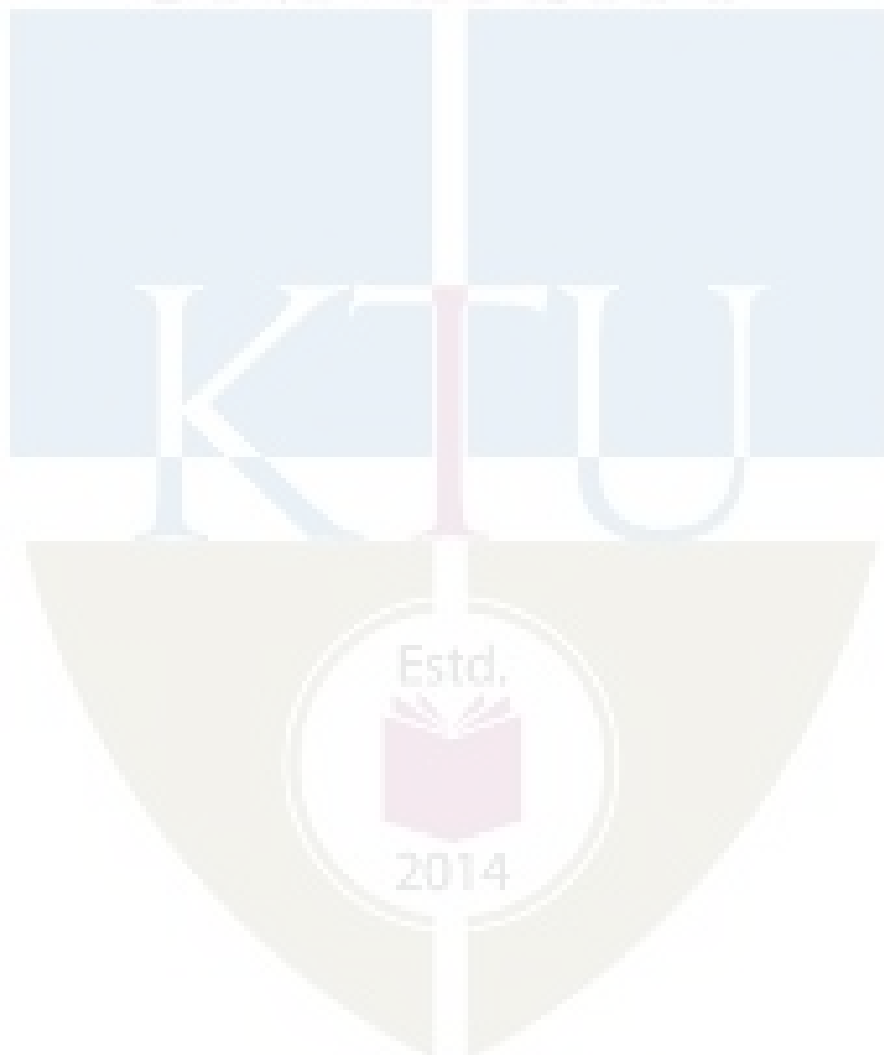
## Reference Books

1. P. Kaye, R. Laflamme, and M. Mosca. An Introduction to Quantum Computing. Oxford, 2007.
2. Eleanor G. Rieffel, Wolfgang H. Polak, “Quantum Computing: A Gentle Introduction,” MIT Press, 2011.
3. Noson Yanofsky and Mirco Mannucci, “Quantum Computing for Computer Scientists”, Cambridge University Press, 2008.
4. Abhijith, J., Adedoyin, Adetokunbo, Ambrosiano, John (and 30 others), “Quantum Algorithm Implementations for Beginners”, [arXiv:1804.03719](https://arxiv.org/abs/1804.03719), 2020.

## Course Contents and Lecture Schedule

No	Topic	No. of Lectures
<b>1</b>	<b>Basics of Linear Algebra</b>	
1.1	History and Overview of Quantum Computation and Quantum Information	1
1.2	Linear Algebra Basics	1
1.3	Linear Operators and matrices	1
1.4	Pauli matrices	1
1.5	Inner Products, Eigen values and Eigen vectors	1
1.6	Hermitian operators and Adjoints, Spectral theorem	2
1.7	Tensor Products	2
<b>2</b>	<b>Basics of Quantum Mechanics</b>	
2.1	State Space Representation - Bloch Sphere	1
2.2	State Evolution – Unitary transformation	2
2.3	Quantum measurement – Projective measurements	2
2.4	Composite systems - Superposition	2
<b>3</b>	<b>Quantum Gates and Circuits</b>	
3.1	Quantum gates – Hadamard gate, NOT gate, controlled-NOT gate, Toffoli gate	2
3.2	Realisation of classical gates with quantum gates – Z Gate, Fredkin Gate	2
3.3	Pauli Matrices – Controlled Swap and Controlled U-operations	2
3.4	Circuit Identities	1
<b>4</b>	<b>Quantum Measurement</b>	
4.1	Basic principle of quantum measurement - Principle of deferred measurement, Principle of implicit measurement	1
4.2	Gates with projective measurements	2
4.3	Universal quantum gates	1

4.4	Universality of two level unitary gates	2
<b>5</b>	<b>Algorithms</b>	
5.1	Quantum Fourier Transform (QFT)	1
5.2	Quantum circuit for QFT Quantum phase estimation	2
5.3	Modular exponentiation	1
5.4	Order finding and factorisation – Deutsch's algorithm	2



APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SIXTH SEMESTER B.TECH DEGREE EXAMINATION, (Model Question Paper)

Course Code: ECT372

Course Name: QUANTUM COMPUTING

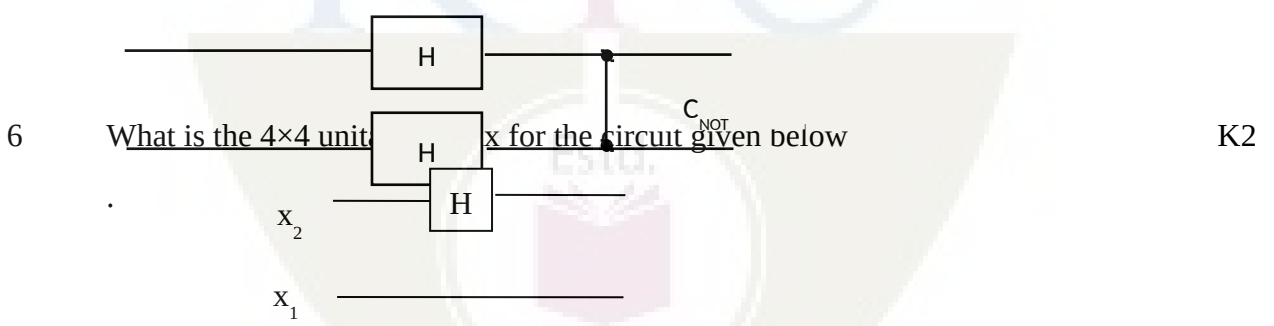
Max. Marks: 100

Duration: 3 Hours

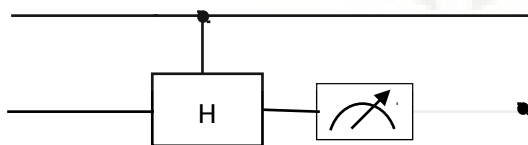
PART A

Answer ALL Questions. Each Carries 3 mark.

- 1 Consider the operator from  $\mathbb{C}^2 \rightarrow \mathbb{C}^2$  given by  $T(x,y) = (ix, iy)$ , where  $Z^2 = -1$ . Find the matrix representation of the Transformation. K2
- 2 Write the Eigen values and Eigen vectors of the matrix  $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ . Is this operator Hermitian? K1
- 3 Write down the Bloch sphere representation of the Quantum bit  $\frac{1}{2}|0\rangle + \frac{j}{\sqrt{2}}|1\rangle$ . K1
- 4 Suppose the first bit of a two bit Quantum System whose state given by  $\alpha|00\rangle + \beta|01\rangle + \chi|10\rangle + \delta|11\rangle$  is measured
  - a. What is the probability that the first bit is observed to be 0?
  - b. Suppose that the first bit is observed to be 0, then what is the resultant state of the system?K3
5. Compute the resultant state of the given circuit for input state  $|00\rangle$ . K3



- 7 State the two basic principles of quantum measurement and explain it's uses. K1
- 8 For the given circuit, 0 was observed by measuring the second bit. What is the resultant Quantum State of the first bit? K3



- 9 Give a decomposition of the controlled-Rk gate into single qubit and  $C_{NOT}$  gates. K2
- 10 Draw the 3 input Quantum Fourier Transform (QFT) circuit. K2

Answer one question from each module; each question carries 14 marks.

**Module - I**

- 11 a. Find the eigenvectors and eigenvalues of the following four matrices: 7

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

CO1

K2

- b. Give the eigenvalues and eigenvectors of this matrix 7

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

CO1

K3

**OR**

- 12 a. A matrix M is Hermitian if  $M^\dagger = M$ . Let M be Hermitian. 7

i. Prove that all of its eigenvalues are real. CO1

ii. Prove that  $v^\dagger M v$  is real, for all vectors v. When  $v^\dagger M v > 0$ , we say that  $M > 0$ . K3

- b. Let M be Hermitian, and define 7

$$U = e^{iM} = \sum_k \frac{(iM)^k}{k!}$$

CO1

Prove that  $U^\dagger U = I$ , where I is the identity matrix. For matrix M, let  $M^\dagger = (M^T)^*$ , where  $M^T$  is the transpose of M, and \* denotes the complex conjugate of M. K3

**Module - II**

- 13 a. What is a Quantum State. Explain with examples 2

CO2

K2

- b. Consider the following two-qubit quantum state,  $|\varphi\rangle$ . 12

$$\frac{\sqrt{2}}{3\sqrt{3}}|00\rangle - \frac{1}{\sqrt{6}}|01\rangle + \frac{2i\sqrt{2}}{3\sqrt{3}}|10\rangle + \frac{5i}{3\sqrt{6}}|11\rangle$$

i. What are the probabilities of outcomes 0 and 1 if the first qubit of  $|\varphi\rangle$  is measured? CO2

ii. What are the probabilities of outcomes 0 and 1 if the second qubit of  $|\varphi\rangle$  is measured? K3

iii. What is the state of the system after the first qubit of  $|\varphi\rangle$  is measured to be a 0?

iv. What is the state of the system if the second qubit of  $|\varphi\rangle$  is measured to be a 1?

v. What are the probabilities of outcomes 0 and 1 if the second qubit of the system is measured, after the first qubit of  $|\varphi\rangle$  has been measured to be 0?

vi. What are the probabilities of outcomes 0 and 1 if the first qubit of the system is measured, after the second qubit of  $|\varphi\rangle$  has been measured to be 1?

14 a. State and explain the four postulates of Quantum Mechanics applied to computing. 8  
CO2

b. Which quantum state do we get if we apply  $(H \otimes I)$  CNOT to  

$$\frac{1}{\sqrt{3}}|00\rangle + \frac{2}{\sqrt{3}}|11\rangle$$
 6  
CO2

Here I is the 1-qubit identity operation, H is the 1-qubit Hadamard, and CNOT is the 2-qubit controlled-not operation with the first (=leftmost) qubit being the control. K3

What is the probability of seeing  $|11\rangle$  if we measure the resulting state in the computational basis?

**Module - III**

15 a. Show that  $XYX = -Y$  and use this to prove that  $X R_y(\theta) X = R_y(-\theta)$ . 7  
CO3

b. An arbitrary single qubit unitary operator can be written in the form 7  
 $U = \exp(i\alpha) R_{\hat{n}}(\theta)$ . for some real numbers  $\alpha$  and  $\theta$ , and a real three-dimensional unit vector  $\hat{n}$ . CO3  
K3

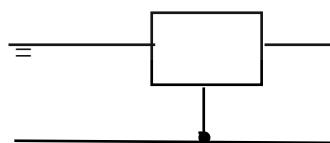
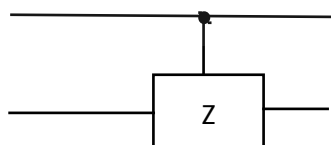
- i. Prove this fact.
- ii. Find values for  $\alpha$ ,  $\theta$ , and  $\hat{n}$  giving the Hadamard gate H.
- iii. Find values for  $\alpha$ ,  $\theta$ , and  $\hat{n}$  giving the phase gate.

**OR**

16 a. It is useful to be able to simplify circuits by inspection, using well-known identities. 7  
Prove the following three identities:

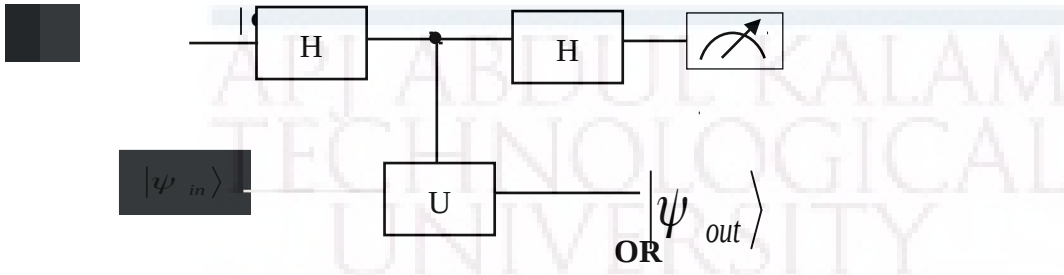
- i.  $HXH = Z$  CO3
- ii.  $HYH = -Y$  K3
- iii.  $HZH = X$

b. Show that 7  
CO3  
K3



**Module - IV**

- 17 Suppose we have a single qubit operator  $U$  with eigenvalues  $\pm 1$ , so that  $U$  is both Hermitian and unitary. Suppose we wish to measure the observable  $U$ . How can this be implemented by a quantum circuit? Show that the following circuit implements a measurement of  $U$ . 14  
CO3  
K3



- 18 a. Derive the circuit implementing the controlled-U operation for an arbitrary single qubit  $U$ , using only single qubit operations and the CNOT gate. 7  
CO3  
K4
- b. Using just CNOTs and Toffoli gates, construct a quantum circuit to perform the transformation given below. 7  
CO3

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

K4

**Module - V**

- 19 a. Derive the circuitry for computing a 4-input Quantum Fourier Transform (QFT). 7  
CO4  
K3
- b. The two qubit Quantum Fourier Transform is given by the following matrix. 7

$$F_2 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

CO4

K3

Sketch a circuit for implementing the operator  $F_2$  using any combination of 1-qubit Hadamard gates; 1-qubit Pauli gates; 2-qubit CNOT gates and controlled phase shifts. Briefly explain your circuit.

- 20 a. Explain the phase estimation algorithm using Quantum Fourier Transform (QFT). 8  
Derive the circuitry for the Quantum Phase estimation. CO4  
K3
- b. Apply Quantum phase estimation to estimate the phase of a T-Gate. 6  
CO4  
K4

